In algebra, we deal with different types of expressions. Grouping them helps us to learn rules and concepts easily. One group of expressions is called polynomials. In a polynomial, the powers are whole numbers. Some special polynomials are named as Monomials, Binomials, and Trinomials.

What is a Monomial?

Monomial is a polynomial which has only one term. Here are some examples.

$$3x$$

$$12axy^2$$

$$-34mnxy^2z$$

$$1.24abx^2y^3$$

Practice 1. Which terms are monomials?

(a) $-1212mn + 32$

(b) 24

(c) $35abc$

(d) $\frac{5xy}{12}$

(e) $21a^2x^4z^6 + 21ax$
Answer. (b), (c), and (d)

Practice 2. Which of the geometrical formulas below are in the form of monomial?

(a) Area of Rectangle

(b) Area of Square

(c) Area of Triangle

(d) Area of Rhombus

(e) Area of Parallelogram

(f) Perimeter of Square

(g) Perimeter of Rectangle

(h) Perimeter of Rhombus

(i) Perimeter of Regular Polygon

Answer. (a), (b), (c), (d), (e), (f), (h), (i)

How to Multiply Monomials?

Any monomial built up of three parts: sing, coefficient, and variables. To multiply two monomials, first multiply their signs. To do this, use the rules of product of signs.

Then multiply the coefficients. Finally multiply the variables. To multiply the variables, use the Product Rule for Exponents.

Product Rule for Exponents

If a and b are real numbers, then for variable x,

$$x^a \times x^b = x^{a+b}$$

For example, to multiply $(-3xy^2)$ by $(-12xy^2z^2)$, follow the steps below.

(a) The product of signs is positive.

(b) The product of coefficients (3 and 12) is 36.

(c) Using the product Rule for Exponents, we get

$$(x) \times (x) = x^2$$

$$y^2 \times y^2 = y^4$$

So, $(-3xy^2)(-12xy^2z^2) = 36x^2y^4z^2$

Practice 3. Multiply.

(a) $3x(xyz)$

(b) $(-21xyz)(3x^2y^3)$

(c) $(11mnx^2)(-2mny^2)$

(d) $(23xyz)(x^2y^3z^4)$

Solution.

(a) $3x(xyz) = 3(x^{1+1})(yz)$

$= 3x^2yz$

(b) $(-21xyz)(3x^2y^3) = (-21 \times 3)(x^{1+2})(y^{1+3})(z)$

$= -63x^3y^4z$

(c) $(11mnx^2)(-2mny^2) = (-11 \times 2)(m^{1+1})(n^{1+1})(x^2y^2)$

$= -22m^2n^2x^2y^2$

(d) $(23xyz)(x^2y^3z^4) = (23)(x^{1+2})(y^{1+3})(z^{1+4})$

$= 23x^3y^4z^5$

Practice 4. Determine the shapes whose areas or perimeters are in the form of monomial.

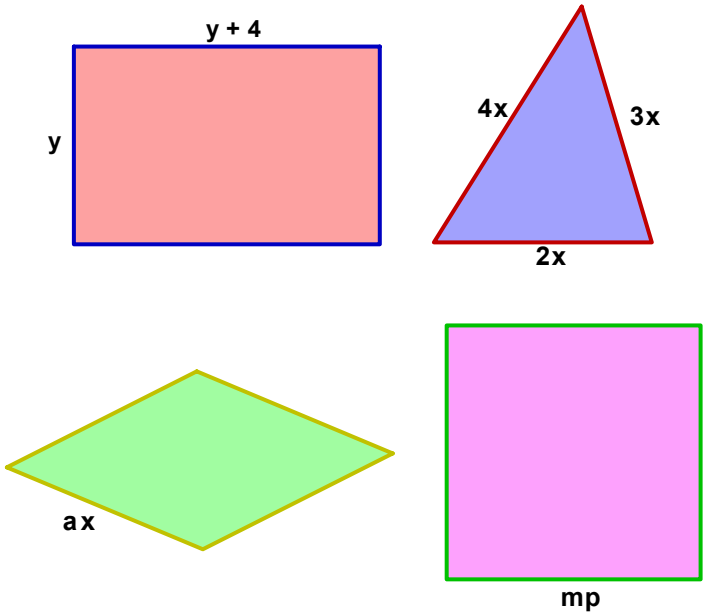

Figure 6.1

Answer. Areas and perimeters of triangle, rhombus, and square

How to Multiply a Monomial by a Binomial?

To multiply a monomial by a binomial, use Distributive Property.

Distributive Property

If a, b, and c are real numbers, then

$a(b + c) = ab + bc$

For example, to multiply $-2xy^2$ by $(x - 3y^2)$ multiply the monomial by each term of the binomial.

$$-2xy^2(x - 3y^2) = -2xy^2(x) - 2xy^2(-3y^2)$$

$$= -2x^2y^2 + 6xy^4$$

Also, you can use the special product identities below to multiply two binomials.

Special Identities

$(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$

$(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Practice 5. Multiply.

(a) $4x(2x + 3xy)$

(b) $-12mn(m + n)$

(c) $21m(mx^2 + mn^2)$

Solution.
(a) $4x(2x + 3xy) = 4x(2x) + 4x(3xy)$

$= 8x^2 + 12x^2y$

(b) $-12mn(m + n) = -12mn(m) - 12mn(n)$

$= -12m^2n - 12mn^2$

(c) $21m(mx^2 + mn^2) = 21m(mx^2) + 21m(mn^2)$

$= 21m^2x^2 + 21m^2n^2$

Practice 6. Use the identities to find each product.

(a) $(a + 2b)^2$

(b) $(m - 3n)^2$

(c) $(x + xy)(x - xy)$

(d) $(1 + 2a)^3$

(e) $(3 - b)^3$

Solution.

(a) $(a + 2b)^2 = a^2 + 2(a)(2b) + (2b)^2$

$= a^2 + 4ab + 4b^2$

(b) $(m - 3n)^2 = m^2 - 2(m)(3n) + (3n)^2$

$= m^2 - 6mn + 9n^2$

(c) $(x + xy)(x - xy) = x^2 - (xy)^2$

$= x^2 - x^2y^2$

(d) $(1 + 2a)^3 = 1^3 + 3(1^2)(2a) + 3(1)(2a)^2 + (2a)^3$

$= 1 + 6a + 12a^2 + 8a^3$

(e) $(3 - b)^3 = 3^3 - 3(3^2)(b) + 3(3)(b^2) - b^3$

$= 27 - 27b + 9b^2 - b^3$

Geometric Application. Find the area of each shape.

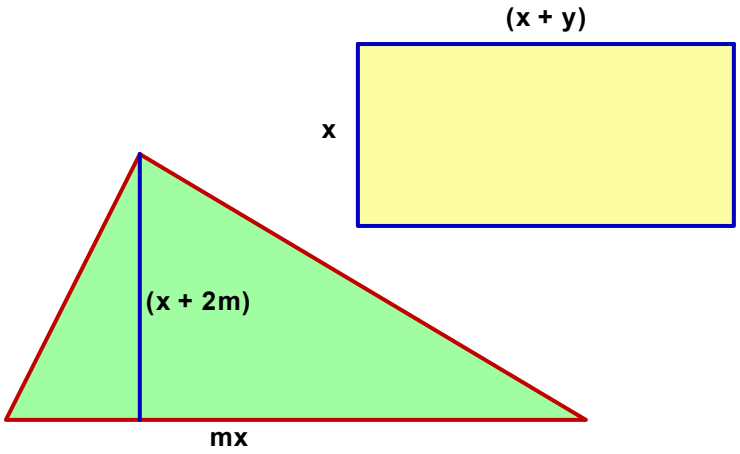

Figure 6.2

Solution.
Area of Rectangle = $x(x + y)$

$= x^2 + xy$

Area of Triangle = $\frac{1}{2}(mx)(x + 2m)$

$= \frac{1}{2}(mx)(x) + \frac{1}{2}(mx)(2m)$

$= \frac{1}{2}mx^2 + m^2x$

Practical Exercise 1. Perform indicated operations.

(a) $-3x(x^3y^3)$

(b) $12mn^3(x^3y)$

(c) $(m + 2n)^2$

(d) $(2m - n)^2$

(e) $(a + ab)(a - ab)$

(f) $(2m - 3n)^3$

(g) $(a + 2b)^3$

Answer.
(a) $-3x^4y^3$
(b) $12mn^3x^3y$
(c) $m^2 + 4mn + 4n^2$
(d) $4m^2 - 4mn + n^2$
(e) $a^2 - a^2b^2$
(f) $8m^3 - 36m^2n + 54mn^2 - 27n^3$
(g) $a^3 + 6a^2b + 12ab^2 + 8b^3$