Polynomials are algebraic expressions made up of several terms. In any polynomial, the exponents of variables must be whole numbers. Here are some examples.

$$-4m^3b + 2n^3b^2 - a^2b^4 - ab$$

$$-x^2y^3z^4 + 2ayz^4 - 4xyz - x^4y^5z$$

$$4mn^3 - 3x^3 - 4mn^3 - 5y^2x^2 - mny^3z^4$$

Opposite of a Polynomial

To find the opposite of a polynomial, simply reverse the sign of each term. For example, the opposite of $-3x^5 - 3x^4 - 2x^2 - 6x - 5$ is $3x^5 + 3x^4 + 2x^2 + 6x + 5$ with all positive terms.

Practice 11. Find opposite of each polynomial.

(a) $31x^5 - 3x^3 + 25x^2 - 46x + 45$

(b) $-56x^8 - 3x^7 + 29x^5 + 65x^3 - 43x$

(c) $x^7 + 33x^5 + 24x^4 - 61x^2 - 35$

Solution.
(a) $-31x^5 + 3x^3 - 25x^2 + 46x - 45$

(b) $56x^8 + 3x^7 - 29x^5 - 65x^3 + 43x$

(c) $-x^7 - 33x^5 - 24x^4 + 61x^2 + 35$

Practice 12. Identify each expression as monomial, binomial, trinomial, or polynomial.

(a) $3x + xyz$

(b) $11x^2y^3$

(c) $321mnx^2 - 2mny$

(d) $xyz - x^2y^3z^4 + 23ay^2z^4 - 43xyz - x^2y^3z^6$

(e) $3xyz - 3xz$

(f) $-22xy + z\,3x^2 - y^3$

(g) $10mnx^2 - 12m + ny^2$

(h) $-3xyz$

(i) $21x + yz - 3x^2 + y^3\,x^2 + 32y - 3z^4$

(j) $mn - x^2 - 2mn - y^2x^2 - y^3z^4$

Answer.
(a) Binomial
(b) Monomial
(c) Binomial
(d) Polynomial
(e) Binomial
(f) Trinomial
(g) Trinomial
(h) Monomial
(i) Polynomial
(j) Polynomial

Practice 13. List geometric formulas in which a monomial is multiplied by a binomial.

Answer.

Area of Trapezoid = $\frac{1}{2}h(a + b)$

Perimeter of Rectangle = $2(a + b)$

Perimeter of Parallelogram = $2(a + b)$

How to Multiply Two Polynomials?

To multiply two polynomials, use the Distributive Property several times. Each time take a term from one of the polynomial and multiply it by all the terms of the other polynomial.

For example, if the first polynomial has 5 terms, we must multiply its terms one by one by the other polynomial. In this case, we will use the Distributive Property five times.

Example. Multiply

$$(3x^3 - 5x^2 - 2x + 5)(x^4 + 2x^3 - x^2 + 7x - 2)$$

Multiply each term of the first polynomial by the other polynomial first.

$$(3x^3 - 5x^2 - 2x + 5)(x^4 + 2x^3 - x^2 + 7x - 2) = (3x^3)(x^4 + 2x^3 - x^2 + 7x - 2)$$
$$+ (-5x^2)(x^4 + 2x^3 - x^2 + 7x - 2)$$
$$+ (-2x)(x^4 + 2x^3 - x^2 + 7x - 2)$$
$$+ (5)(x^4 + 2x^3 - x^2 + 7x - 2)$$

Use Distributive Property to find the product of monomials by polynomials.

$$(3x^3 - 5x^2 - 2x + 5)(x^4 + 2x^3 - x^2 + 7x - 2) =$$

$$(3x^3)(x^4) + (3x^3)(2x^3) + (3x^3)(-x^2) + (3x^3)(7x) + (3x^3)(-2) =$$

$$(-5x^2)(x^4) + (-5x^2)(2x^3) + (-5x^2)(-x^2) + (-5x^2)(7x) + (-5x^2)(-2) +$$

$$(-2x)(x^4) + (-2x)(2x^3) + (-2x)(-x^2) + (-2x)(7x) + (-2x)(-2) +$$

$$(5)(x^4) + (5)(2x^3) + (5)(-x^2) + (5)(7x) + (5)(-2)$$

Now, use the rule of multiplying a monomial by a monomial.

$$(3x^3 - 5x^2 - 2x + 5)(x^4 + 2x^3 - x^2 + 7x - 2) =$$

$$3x^7 + 6x^6 - 3x^5 + 21x^4 - 6x^3 - 5x^6 - 10x^5 + 5x^4 - 35x^3 + 10x^2$$

$-2x^5 - 4x^4 + 2x^3 - 14x^2 + 4x + 5x^4 + 10x^3 - 5x^2 + 35x - 10$

Combine like terms.

$(3x^3 - 5x^2 - 2x + 5)(x^4 + 2x^3 - x^2 + 7x - 2) =$

$= 3x^7 + x^6 - 15x^5 + 27x^4 - 29x^3 - 9x^2 + 39x - 10$

Practice 14. Find a polynomial representing the area of the rectangle below.

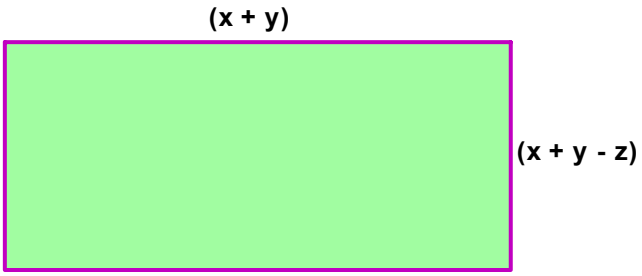

Figure 6.4

Solution.
Area of Rectangle = $(x + y)(x + y - z)$

$= x(x + y - z) + y(x + y - z)$

$= x^2 + xy - xz + xy + y^2 - yz$

$= x^2 + 2xy - xz + y^2 - yz$

Practical Exercise 4. Find each product.

(a) $(x^2 + x + 1)(x^4 + x^3 - x^2 + 1)$

(b) $(mn - m - n + 1)(m + n + 1)$

Answer.
(a) $x^6 + 2x^5 + x^4 + x + 1$
(b) $m^2n + mn^2 - m^2 - n^2 - mn + 1$